Linear Stability Analysis of Thermo-Solutal Couple-Stress Fluid Flow with Linear Heating in Porous Medium

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Abstract—*The onset of double-diffusive convection in a couple-stress fluid-saturated with horizontal porous layer is analysed by using linear and weak nonlinear stability analyses. It is obtained that the couple-stress parameter and the solute Rayleigh number have a stabilizing effect on stationary, oscillatory and finite-amplitude convection. The heat and mass transfer decreases with an increase in the values of couple-stress parameter and diffusivity ratio, while both increase with an increase in the value of the solute Rayleigh number.*

1. INTRODUCTION

The study of convective flow of thermo-solutal couple-stress fluid in darcy porous medium with heat and mass transfer under the influence of chemical reaction with heat source has practical applications in many areas of science and engineering. Natural convection flows occur frequently in nature due to temperature differences, concentration differences, and also due to combined effects. The concentration difference may sometimes produce qualitative changes to the rate of heat transfer. Recently, the equally problem of hydromagnetic convective flow of a conducting fluid through a porous medium has been investigated.

Many important developments in literature of stability theory are given by, Chandrasekhar(1981), Nield and Bejan (2012). Bhadauria et al. (2012) has made the stability analysis of convection in a binary fluid-saturated horizontal porous layer with internal heat source. Recently, viscoelastic fluid flow in porous media has attracted considerable attention, due to the large demands of such diverse fields as biorheology, geophysics, chemical industries, and petroleum industries. Also Bhadauria group (2012),(2013) have studied the problem of thermal instability in porous media with internal heating, considering various physical models. Cimpean (2012), analyzes the mixed convection flow of a nanofluid in an inclined channel filled with a porus medium. The main focus was on the effects of the main parameters, such as solid volume fraction of the nanoparticles, the mixed convection parameter, the Péclet number and the inclination of the channel to the horizontal, on the thermal performances of the flow. Gaikwad and Kamble (2012) have investigated the Soret effect on double diffusive convection in a horizontal sparsely

packed porous layer. Narayana et al. (2012) studied the linear and weakly nonlinear stability analysis of double-diffusive convection in a porous medium saturated by a Maxwell fluid in the presence of cross diffusion effects. The effects of the Soret and Dufour parameters on the onset of double diffusive convection in a Maxwell fluid are investigated under the assumption of a single phase model with local thermal equilibrium (LTE) between the porous matrix and the Maxwell fluid. Harfash (2013) studied double-diffusive convection in a reacting fluid with a concentration and magnetic field effect–based internal heat source by using linear instability analysis and nonlinear stability analysis and using the finite element method of p order. Further Nygard et al. (2013) done a computational study on turbulent flow through an abrupt axisymmetric contraction. Rana (2014) studied the thermal convection in couple-stress fluid in hydromagnetics saturating a porous medium and found that couple-stress parameter has stabilizing effect on the system. The onset of convection in a horizontal layer heated from below (Bénard problem) for a nanofluid was studied by Rana et al. (2014). Kumar et al.(2015) investigated the thermosolutal convection in a viscoelastic dusty fluid with hall currents in porous medium. Kumar et al.(2016) studied the effects of horizontal magnetic field and rotation on thermal instability of a couple-stress fluid through a porous medium. Singh et al.(2016) analysed the the transport of vorticity in magnetic Maxwellian viscoelastic fluid -particle mixtures in porous medium. Chand et al.(2017), investigated the thermal instability in a layer of couple stress nanofluid saturated porous medium and also studied the thermal instability in a horizontal layer of Couple-stress nanofluid in a porous medium for more realistic boundary conditions. Kumar et al.(2017) studied the effect of horizontal magnetic field and horizontal rotation on thermo-solutal stability of a dusty couple-stress fluid through a porous medium: a brinkman model. Rana et al.(2018) studied the stability analysis of double-diffusive convection in a couple stress nanofluid. Singh M.(2018), investigated the double-diffusive convection of synovial (couple-stress) fluid in the presence of hall current through a porous medium and studied the effect of Hall

current on thermosolutal convection of a couple-stress fluid through porous medium. Current paper is a review work based on Malashetty et al.(2010) analysed Double-diffusive convection in a Darcy porous medium saturated with a couple-stress fluid.

2. MATHEMATICAL FORMULATION AND STABILITY ANALYSIS

Let's consider a horizontal porous layer which is saturated with a couple-stress fluid between two parallel infinite stress-free boundaries, $z = 0, d$, heated from below. The temperature and concentration differences between the planes are respectively ΔT and ΔS. The z-axis is taken vertically upward in the gravitational field in a Cartesian frame of reference. Let's assume that Oberbeck–Boussinesq approximation is true and the flow in the porous medium is carried out by the modified Darcy's law. The study of double-diffusive convection in a couple-stress fluid-saturated horizontal porous layer with the basic equations are given by:

$$\nabla \cdot q = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon} q \cdot \nabla q\right) = -\nabla p + \rho g - \frac{1}{k}(\mu - \mu_c \nabla^2) q, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (q \cdot \nabla)T = \kappa_T \nabla^2 T + Q(T - T_0), \tag{3}$$

$$\varepsilon \frac{\partial S}{\partial t} + (q \cdot \nabla)S = \kappa_S \nabla^2 S, \tag{4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)], \tag{5}$$

where $q = (u, v, w)$ is the velocity; p is the pressure; ρ is the density; T is the temperature; S is the solute concentration; Q is heat source parameter; T_0, S_0 and ρ_0 are the reference temperature, concentration and density, respectively; the acceleration due to gravity is given by g; k is the permeability of the porous medium; ε is the porosity; μ is the fluid viscosity; μ_c is the couple-stress viscosity; β_T and β_S are the thermal expansion coefficient and the solute expansion coefficient, respectively;

and k_T and k_S are the effective thermal diffusivity and the solute diffusivity, respectively. Moreover,

$$\gamma = \frac{(\rho c)_m}{(\rho c_p)_f}, \kappa_T = \frac{(1-\varepsilon)K_S + \varepsilon K_f}{(\rho c_p)_f}, (\rho c)_m = (1 - \varepsilon)(\rho c)_s + \varepsilon(\rho c_p)_f.$$

Here, c_p is the specific heat of the fluid at constant pressure; c is the specific heat of the solid; K is taken as the thermal conductivity; and the subscripts f, s and m denote fluid, solid and porous medium values, respectively. The basic state of the fluid is considered to be quiescent and is given by: $q_b = (0,0,0), p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z)$ (6)

The solute concentration $S_b(z)$, temperature $T_b(z)$, pressure $p_b(z)$ and density $\rho_b(z)$ satisfy the equations as follows:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{7}$$

$$\frac{d^2 T_b}{dz^2} = 0, \tag{8}$$

$$\frac{d^2 S_b}{dz^2} = 0, \tag{9}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)] \tag{10}$$

On the basic state, we consider perturbations in the following form:

$$q = q_b + q'(x, y, z, t), T = T_b(z) + T'(x, y, z, t), S = S_b(z) + S'(x, y, z, t), p = p_b(z) + p'(x, y, z, t), \rho = \rho_b(z) + \rho'(x, y, z, t), \tag{11}$$

where primes indicate perturbations. Introducing (11) in equation (1)–(5) and using basic state equations, we obtain

$$\nabla \cdot q' = 0, \tag{12}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial q'}{\partial t} + \frac{1}{\varepsilon} q' \cdot \nabla q'\right) = -\nabla p' + \rho' g - \frac{\mu}{k}\left(1 - \frac{\mu_c}{\mu}\nabla^2\right)q' \tag{13}$$

$$\gamma \frac{\partial T'}{\partial t} + (q' \cdot \nabla)T' + w'\frac{\partial T_b}{\partial z} = \kappa_T \nabla^2 T' + QT' \tag{14}$$

$$\varepsilon \frac{\partial S'}{\partial t} + (q' \cdot \nabla)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S \nabla^2 S', \tag{15}$$

$$\rho' = -\rho_0(\beta_T T' - \beta_S S') \tag{16}$$

We consider only two-dimensional disturbances and define ψ as stream function which is given by

$$(u', w') = \left(-\frac{\partial \psi}{\partial z}, \frac{\partial \psi}{\partial x}\right) \tag{17}$$

which also satisfy the continuity equation (12). Now let's eliminate the pressure term from Eq.(13) by introducing the stream function ψ, and non-dimensionalising the resulting equation as well as equation(14) and (15), considering the following non-dimensional parameters,

$$(x^*, z^*) = \left(\frac{x}{d}, \frac{z}{d}\right), t^* = t\left(\frac{\gamma d^2}{\kappa_T}\right), \psi^* = \frac{\psi}{\kappa_T}, T^* = \frac{T'}{\Delta T}, S^* = \frac{S'}{\Delta S}, \tag{18}$$

We obtain

$$\left(\frac{1}{\gamma V_a}\frac{\partial}{\partial t} + 1 - C\nabla^2\right)\nabla^2 \psi - \frac{1}{V_a}\left(\frac{\partial \psi}{\partial x}\frac{\partial(\nabla^2 \psi)}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial(\nabla^2 \psi)}{\partial x}\right) = -Ra_T \frac{\partial T}{\partial x} + Ra_S \frac{\partial S}{\partial x}, \tag{19}$$

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} - \left(\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial T}{\partial x}\right) - \nabla^2 T - QT' = 0, \tag{20}$$

$$\frac{\varepsilon}{\gamma}\frac{\partial S}{\partial t} + \frac{\partial \psi}{\partial x} - \left(\frac{\partial \psi}{\partial x}\frac{\partial S}{\partial z} - \frac{\partial \psi}{\partial z}\frac{\partial S}{\partial x}\right) - \tau\nabla^2 S = 0. \tag{21}$$

Here $V_a = \frac{\varepsilon v d^2}{\kappa_T k}$, $C = \frac{\mu_c}{\mu d^2}$, $\tau = \frac{\kappa_S}{\kappa_T}$, $Ra_T = \frac{\beta_T g \Delta T dk}{v \kappa_T}$, $Ra_S = \frac{\beta_S g \Delta S dk}{v \kappa_S}$.

The dimensionless groups which appear are Vadasz number V_a, thermal Rayleigh number Ra_T, solute Rayleigh number Ra_S, couple-stress parameter C and diffusivity ratio τ. The

asterisks have been dropped for simplicity. Further, to restrict the number of parameters, let's set ε and γ equal to unity. Equations (19)–(21) get solved for stress-free, isothermal, vanishing couple-stress boundary conditions, namely

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = S = Q = 0, \text{ at } z = 0, 1. \tag{22}$$

3. METHOD OF SOLUTION

We discuss in this section the linear stability analysis, which is very useful in the local non-linear stability analysis discussed in the proceeding section. For this study, the Jacobians in Equation (19)–(21) are neglected and suppose that the solutions to be periodic waves of the form

$$\begin{pmatrix} \psi \\ T \\ S \end{pmatrix} = e^{\sigma t} \begin{pmatrix} \psi_0 \sin n\pi\alpha x \\ \theta_0 \cos n\pi\alpha x \\ \Phi_0 \cos n\pi\alpha x \end{pmatrix} \sin(n\pi z)(n = 1,2,3,\dots) \tag{23}$$

where σ is the growth rate and $\sigma = \sigma_r + i\sigma_i$. α is the horizontal wavenumber. Substituting equation (23) in equations (19)–(21), we get

$$\left(\frac{\sigma}{V_a} + \eta\right) \delta_n^2 \psi_0 = n\pi\alpha(-Ra_T\theta_0 + Ra_S\Phi_0) \tag{24}$$

$$(\delta_n^2 + \sigma)\theta_0 = -n\pi\alpha\psi_0 \tag{25}$$

$$(\tau\delta_n^2 + \sigma)\Phi_0 = -n\pi\alpha\psi_0, \tag{26}$$

where $\delta_n^2 = n^2\pi^2(1 + \alpha^2), \eta = 1 + C\delta_n^2$.

The parameter η is the couplestress viscosity of the fluid. In the case of Newtonian fluid, we have $\eta = 1$. Now, equations (24)–(26) can be written in matrix form as

$$AX = 0, \tag{27}$$

where

$$A = \begin{pmatrix} \left(\frac{\sigma}{V_a} + \eta\right)\delta_n^2 & Ra_T n\pi\alpha & -Ra_S n\pi\alpha \\ n\pi\alpha & \sigma + \delta_n^2 & 0 \\ n\pi\alpha & 0 & \sigma + \tau\delta_n^2 \end{pmatrix}, \ X = \begin{pmatrix} \psi_0 \\ \theta_0 \\ \Phi_0 \end{pmatrix} \text{ and }$$

$$0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

For non-trivial solution for X, the determinant of the matrix A to be vanished, which gives

$$Ra_T = \frac{(\sigma+\delta_n^2)(\sigma+\tau\delta_n^2)\left(\frac{\sigma}{V_a}+\eta\right)\delta_n^2 + Ra_S n^2\pi^2\alpha^2(\sigma+\delta_n^2)}{n^2\pi^2\alpha^2(\sigma+\tau\delta_n^2)} . \tag{28}$$

Normally, we assume that for $n = 1$ which is the most unstable mode (fundamental mode). Accordingly, we set $\eta = 1 + C\delta^2, \delta^2 = \pi^2(\alpha^2 + 1)$, in our further study. For the steady case, we have $\sigma = 0$ at the marginal stability. Then, the Rayleigh number becomes

$$Ra_T^{st} = \frac{\eta\delta^4}{\pi^2\alpha^2} + \frac{Ra_S}{\tau} \tag{29}$$

The minimum value of the Rayleigh number Ra_T^{st} appears at the critical wavenumber $\alpha = \alpha_c$, and α_c satisfies the following equation

$$2C\pi^2(\alpha^2)^2 + (1 + C\pi^2)\alpha^2 - (1 + C\pi^2) = 0 \tag{30}$$

Here the critical wavenumber α_c depends on the couple-stress parameter C. In the case of the single-component system, $Ra_S = 0$, equation (29) becomes

$$Ra_T^{st} = \frac{(1+C\delta^2)\delta^4}{\pi^2\alpha^2}, \tag{31}$$

In the presence of couple stresses, equation (31) gives rise to the critical value of the Rayleigh number

$$Ra_{T,c}^{st} = \frac{\pi^2(1+\alpha_c^2)[1+C\pi^2(1+\alpha_c^2)]}{\alpha_c^2} \tag{32}$$

The critical wavenumber α_c can be obtained from equation (30). In the absence of couple stresses, i.e. when $C = 0$, equation (32) gives

$$Ra_T^{st} = \frac{\pi^2(1+\alpha^2)^2}{\alpha^2} \tag{33}$$

with critical values when $\alpha_c = 1$ and $Ra_T^{st} = 4\pi^2$ for Newtonian fluid through a Darcy porous layer heated and salted from below.

Now $\sigma = i\omega$ (ω is real) in Equation (28) and also rearranging the terms we get the oscillatory Rayleigh number Ra_T^{osc} at the margin of the stability, in the form

$$Ra_T^{osc} = \frac{(1+\tau)\left[\eta\delta^4(1+\tau) + \frac{\tau\delta^6}{V_a} + \eta^2 V_a\delta^2\right] + Ra_S\pi^2\alpha^2\left(2+\tau+\frac{\eta V_a}{\delta^2}\right)}{\pi^2\alpha^2\left(2\tau+1+\frac{\eta V_a}{\delta^2}\right)} \tag{34}$$

with the non-dimensional frequency ω^2 and

$$\omega^2 = -\frac{\delta^4\tau^2\left(\frac{\tau^2}{V_a}+\eta\right) + Ra_S\pi^2\alpha^2(\tau-1)}{\eta + \left(\delta^2/V_a\right)} \tag{35}$$

A careful observation at the expression for frequency ω reveals that oscillatory convection is possible only if $\tau < 1$.

Let's carry out the Finite-amplitude analysis in this section by Fourier series representation for stream function ψ, temperature T and concentration S in the form

$$\psi = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t) \sin(m\pi\alpha x) \sin(n\pi z), \tag{36}$$

$$T = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} B_{mn}(t) \cos(m\pi\alpha x) \sin(n\pi z), \tag{37}$$

$$S = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} E_{mn}(t) \cos(m\pi\alpha x) \sin(n\pi z). \tag{38}$$

Substituting equations(36)–(38) in(19)–(21), we get a system of coupled, nonlinear ordinary differential equations.

The first effect of nonlinearity is to distort the temperature and concentration fields by the interaction of ψ, T and S. The distortion of these fields gives rise to a change in the horizontal mean, i.e. a component of the form $\sin(2\pi z)$ will be formed. Therefore, a minimal Fourier series that explains the finite-amplitude double-diffusive convection is given by

$$\psi = A_1(t)\, sin(\pi\alpha x)sin(\pi z), \tag{39}$$

$$T = B_1(t)\, cos(\pi\alpha x)sin(\pi z) + B_2(t)\, sin(2\pi z), \tag{40}$$

$$S = E_1(t)\, cos(\pi\alpha x)sin(\pi z) + E_2(t)\, sin(2\pi z), \tag{41}$$

Putting equations (39)–(41) into equations (19)–(21) and equating the coefficients of like terms, the following nonlinear autonomous system of differential equations can be obtained:

$$\dot{A}_1 = -\frac{Ra_T \pi\alpha V_a}{\delta^2} B + \frac{Ra_S \pi\alpha V_a}{\delta^2} E_1 - \delta^2 \eta V_a A_1, \tag{42}$$

$$\dot{B}_1 = -\pi\alpha A_1 - \delta^2 B_1 - \pi^2\alpha A_1 B_2, \tag{43}$$

$$\dot{B}_2 = -4\pi^2 B_2 + \frac{\pi^2\alpha}{2} A_1 B_1, \tag{44}$$

$$\dot{E}_1 = -\pi\alpha A_1 - \delta^2 \tau E_1 - \pi^2\alpha A_1 E_2, \tag{45}$$

$$\dot{E}_2 = -4\pi^2 \tau E_2 + \frac{\pi^2\alpha}{2} A_1 E_1, \tag{46}$$

where the overdot represents the derivative with respect to time.

Like the original equations (12)–(16), equations (42)–(46) must be dissipative. Therefore, volume in the phase space must contract. To prove volume contraction, we must prove that the velocity field has a constant negative divergence. Indeed,

$$\frac{\partial \dot{A}_1}{\partial A_1} + \frac{\partial \dot{B}_1}{\partial B_1} + \frac{\partial \dot{B}_2}{\partial B_2} + \frac{\partial \dot{E}_1}{\partial E_1} + \frac{\partial \dot{E}_2}{\partial E_2} = -[\delta^2(\eta V_a + 1 + \tau) + 4\pi^2(1 + \tau^2)]. \tag{47}$$

From equation(47) we conclude that if a set of initial points in phase space occupies a region $V(0)$ at time $t = 0$, then after some time t, the end points of the corresponding trajectories will occupies a volume

$$V(t) = V(0)exp[-\{\delta^2(\eta V_a + 1 + \tau) + 4\pi^2(1 + \tau^2)\}t]$$

We note that the system of equations (4.7)–(4.11) is invariant under symmetry transformation $(A_1, B_1, B_2, E_1, E_2) \rightarrow (-A_1, -B_1, -B_2, -E_1, -E_2)$.

Setting the left-hand sides of equations (42)–(46) equal to zero, we obtain

$$\eta\delta^4 A_1 + Ra_T \pi\alpha B_1 - Ra_S \pi\alpha E_1 = 0, \tag{48}$$

$$\pi\alpha A_1 + \delta^2 B_1 + \pi^2\alpha A_1 B_2 = 0, \tag{49}$$

$$8B_2 - \alpha A_1 B_1 = 0, \tag{50}$$

$$\pi\alpha A_1 + \tau\delta^2 E_1 + \pi^2\alpha A_1 E_2 = 0, \tag{51}$$

$$8\tau E_2 - \alpha A_1 E_1 = 0. \tag{52}$$

Eliminating all coefficients except A_1 between equations (48)–(52), we obtain

$$a_1 x^2 + b_1 x + c = 0, \tag{53}$$

where $x = \frac{A_1^2}{8}$, $a_1 = \eta\delta^2\alpha^4$, $b_1 = \frac{\eta\delta^4\alpha^2}{\pi^2}(1 + \tau^2) + \frac{\alpha^4}{\delta^2}(\tau Ra_S - Ra_T)$, $c_1 = \frac{\eta\delta^6\tau^2}{\pi^4} + \frac{\tau^2\alpha^2}{\pi^2}\left(\frac{Ra_S}{\tau} + Ra_T\right)$.

The finite-amplitude Rayleigh number can be written in the form

$$Ra_T^f = \frac{1}{2x_1}\left\{-x_2 + (x_2^2 - 4x_1 x_3)^{(1/2)}\right\}, \tag{54}$$

where $x_1 = \frac{\alpha^8}{\delta^4}$, $x_2 = \frac{4\eta\delta^2\alpha^6\tau^2}{\pi^2} - \frac{2\eta\delta^2\alpha^6(1+\tau^2)}{\pi^2} - \frac{2\alpha^8\tau Ra_S}{\delta^4}$,

$$x_3 = \frac{\eta^2\delta^8\alpha^4}{\pi^4}(1 + \tau^2 + \tau^4) + \frac{\alpha^8\tau^2 Ra_S^2}{\delta^4}$$
$$+ \frac{2\eta\delta^2\alpha^6\tau(1+\tau^2)Ra_S}{\pi^2} - \frac{4\eta\delta^2\tau\alpha^6 Ra_S}{\pi^2}$$

Let's consider H and J are the rates of heat and mass transport per unit area, respectively as follows:

$$H = -\kappa_T \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0} \tag{55}$$

$$J = -\kappa_S \left\langle \frac{\partial S_{total}}{\partial z} \right\rangle_{z=0} \tag{56}$$

where a horizontal average is denoted by the angular bracket and

$$T_{total} = T_0 - \Delta T \frac{z}{d} + T(x, z, t), \tag{57}$$

$$S_{total} = S_0 - \Delta S \frac{z}{d} + S(x, z, t), \tag{58}$$

Substituting equations (40) and (41) into equations (57) and (58), respectively, and using the resultant equations in (55) and (56), we have

$$H = \frac{\kappa_T \Delta T}{d}(1 - 2\pi B_2) \tag{59}$$

$$J = \frac{\kappa_S \Delta S}{d}(1 - 2\pi E_2) \tag{60}$$

The Nusselt and Sherwood numbers are represented by

$$Nu = \frac{H}{\kappa_T \Delta T/d} = 1 - 2\pi B_2 \tag{61}$$

$$Sh = \frac{J}{\kappa_S \Delta T/d} = 1 - 2\pi E_2 \tag{62}$$

Writing B_2 and E_2 in terms of A_1, using equations (48)–(52), and replacing in equations (61) and (62), respectively, we get

$$Nu = 1 + \frac{2\pi^2\alpha^2 x}{\delta^2 + \pi^2\alpha^2 x} \tag{63}$$

$$Sh = 1 + \frac{2x[Ra_T\pi^2\alpha^2 - \eta\delta^6 + \eta\pi^2\alpha^2 x\delta^4]}{\tau Ra_S(\delta^2 + \pi^2\alpha^2 x)} \tag{64}$$

The second term on the right-hand side of equations (63) and (64) explains the convective contribution to heat and mass transport, respectively.

4. RESULT AND DISCUSSION

The linear theory is explained by considering the usual normal mode technique and the nonlinear theory is by the truncated Fourier series method. Mathematical expressions for the oscillatory, stationary and finite-amplitude Rayleigh numbers for different values of parameters such as diffusivity ratio, couple-stress parameter and solute Rayleigh number are calculated and the results are plotted in figures. Figure 1

displays the neutral stability curves for various values of the couple-stress parameter and for fixed values of $\tau = 0.5, Va = 1.0$, and $Ra_S = 150.0$. The effect of Vadasz number on neutral curves for the oscillatory mode is displayed in figure 2 for fixed values of $Ra_S = 150.0, \tau = 0.5$ and $C = 1.0$. The variation of the critical Rayleigh number for both stationary and oscillatory modes with the solute Rayleigh number for different values of the couple-stress parameter C and fixed values of $Va = 1.0$ and $\tau = 0.5$ is depicted in figure 3.

Figure 4 indicates the variation of the critical Rayleigh number for stationary and oscillatory modes with solute Rayleigh number for different values of diffusivity ratio.
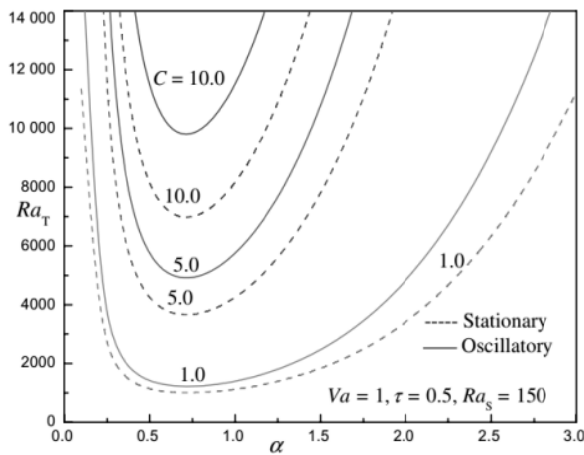

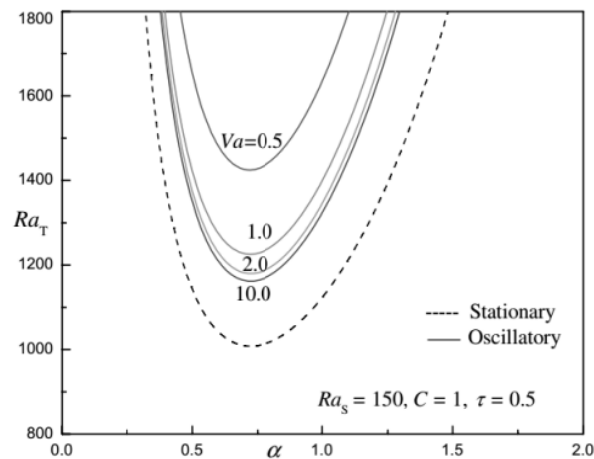**Figure 1. Neutral stability curves for different values of the couple stress parameter C.**

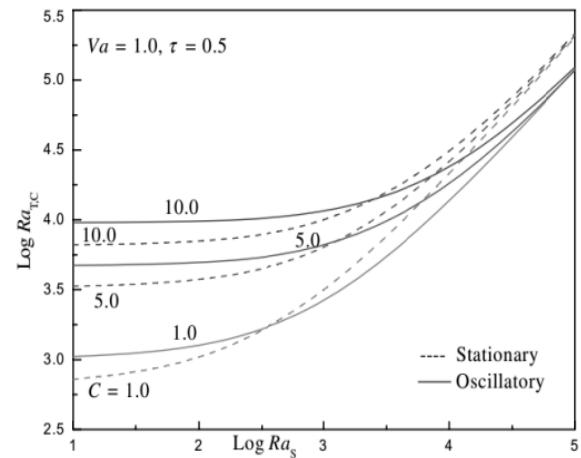**Figure 2. Neutral stability curves for different values of the Vadasz number V_a.**

Figure 3. Variation of the critical Rayleigh number $Ra_{T,C}$ with the solute Rayleigh number Ra_S for different values of C.
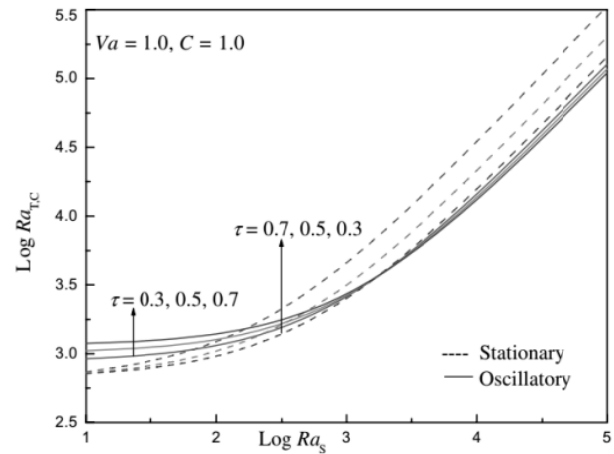

Figure 4. Variation of the critical Rayleigh number $Ra_{T,C}$ with the solute Rayleigh number Ra_S for different values of τ .

5. CONCLUSION & FUTURE SCOPE

The onset of double-diffusive convection in a couple-stress fluid-saturated with a horizontal porous layer is discussed by using linear and weak nonlinear stability analyses. The modified Darcy equation which includes the time derivative term and the inertia term is used to model the momentum equation. The expressions for stationary, oscillatory and finite-amplitude Rayleigh number are found as a function of the governing parameters. The effect of couple-stress parameter, solute Rayleigh number, Vadasz number and diffusivity ratio on stationary, oscillatory and finite-amplitude convection is displayed graphically. The following conclusions are drawn:

1. The solute Rayleigh number and the couple-stress parameter have a stabilizing effect on the oscillatory, stationary, and finite-amplitude convection.

2. The Vadasz number advances the onset of oscillatory convection, showing that it has a destabilizing effect.

3. The diffusivity ratio has a destabilizing effect in the case of stationary and finite amplitude modes, but it has a dual effect in the case of the oscillatory mode depending upon the parameter values.

4. The heat and mass transfer decreases with an increase in the values of couple-stress parameter C and diffusivity ratio τ, but both increase with an increase in the value of the solute Rayleigh number Ra_S.

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